THERMODYNAMIC INSTABILITIES IN MACHINING PROCESSES. 1. GENERAL SOLUTION OF A MATHEMATICAL MODEL

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The author considers a mathematical model that describes the formation of thermodynamic instabilities (outgrowths, adiabatic shifts, and others) in machining processes in the context of mesoscopic, microscopic, and macroscopic approaches to the technological system.

Introduction. One of the topical problems of intensification and automation of mechanical engineering is ensuring stable characteristics for the processes of machining difficult-to-machine materials and stable quality parameters for the produced surfaces.

When machining brittle materials at low speeds the stability of the cutting process is disturbed owing to the formation of a leading crack in the material, which leads to the formation of an elemental chip (Fig. 1) [1]. Intensification of the process by increasing the cutting speed or by additional heating brings the metal to a more plastic state and ensures the formation of a jointed (fragmental) chip. The disturbance of the thermodynamic equilibrium in cutting as a result of the self-organized process of friction leads to thermodynamic hardening of the plastic machined material that forms stagnant dissipative structures in the form of outgrowths on the front surface of a cutting edge [2]. A further increase in the cutting speed does not allow retarded volumes of the machined material to be fixed at the cutting edge, and after the transient pulsating regime of contact interaction over the front and rear surfaces of the cutting edge it ensures stable formation of an overflow chip [3]. At high speeds of cutting plastic materials the temperature weakening of the metal in a narrow localized zone of the most intense deformations leads to loss of stability for the zone of chip formation and as a result of this to localized thermoplastic shifts. Owing to the development of instability under conditions of adiabatic shift a step (cyclic) chip forms [4].

To improve the efficiency and control the machining processes it is possible to use different methods of action on the shaping zone via the machined material, via the tool, and by means of the technological medium. Preheating of the machined material [5], additional travels of the tool's edge [6], and lubricants and coatings on the cutting edge [7] have gained wide use (see Fig. 1).

To study diverse thermodynamic instabilities in machining processes, it is appropriate to consider a mathematical model, analyze it, and compare the obtained numerical solutions with experimental results in machining different materials.

Mathematical Model. In the zone of formation of the surface by machining we consider a field of extensive quantities that describe the state of the system in the context of a mesoscopic approach and compare this description with results obtained in the context of macroscopic and microscopic approaches [8].

An extensive function of the state of the system is described by the expression [9]

$$Z(\tau) = \int_{V} z(\mathbf{r}, \tau) d\mathbf{r}.$$

The general local balance equation of the quantity Z is

$$\partial z (\mathbf{r}, \tau) / \partial \tau + \nabla \cdot \mathbf{F}_z (\mathbf{r}, \tau) = q_z (\mathbf{r}, \tau).$$

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Fig. 1. Types of thermodynamic instabilities in processes of metal cutting. Chips: I) elemental chip, II) jointed chip, III) outflow chip, IV) step chip. Outgrowths: V, VI, VII) of three types. Zones: VIII) pulsating contact zone on the front surface; IX) of relative stagnation; X) of plastic and ductile contacts; XI) of interaction on the rear surface with periodically removed volumes of material on the front surface; XII) pulsating contact zone on the rear surface; XIII) of relative stagnation; XIV) plastic contact one.

In explicit form, balance equations are equations of the hydrodynamic field:

$$\partial \rho / \partial \tau + \nabla \cdot (\rho \mathbf{v}) = 0, \qquad (1)$$

$$\partial (\rho \mathbf{v}) / \partial \tau + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) + \nabla \cdot \mathbf{P}^* = \rho \mathbf{F}^*, \qquad (2)$$

$$\partial (\rho e) / \partial \tau + \nabla \cdot (\rho e \mathbf{v}) + \nabla \cdot \mathbf{F}_q = \mathbf{F}^* \mathbf{F}_d - \mathbf{P}^* \cdot \cdot \nabla \cdot \mathbf{v} \,. \tag{3}$$

By using the balance equations (1)-(3) and the fundamental Gibbs equation $T\delta s = \delta e + P^*\delta(1/\rho)$ we can obtain the equation of the entropy density ρs [9]

$$\partial (\rho s) / \partial \tau + \nabla \cdot (\rho v s) + \nabla \cdot F_s = \sigma^*.$$
 (4)

The entropy production σ^* of (4) enables us to determine the stability condition for stationary states of the open system: $d\sigma^*/d\tau \le 0$ [10]. Consequently, the formation of dissipative structures has special entropy criteria that can be represented, in the context of the macroscopic approach, as physical criteria in particular cases [8].

To consider the field equations in the context of the microscopic approach, it is appropriate to use the random phase function method [11]. The microscopic density in phase space is described by the expression

$$N(\mathbf{r}, \mathbf{p}, \tau) = \sum_{i=1}^{n} \delta(\mathbf{r} - \mathbf{r}_{i}(\tau)) \delta(\mathbf{p} - \mathbf{p}_{i}(\tau))$$

and satisfies the equation

$$\partial N/\partial \tau + (\mathbf{p}/m) \partial N/\partial \mathbf{r} + \mathbf{f} \partial N/\partial \mathbf{p} = \mathbf{0}$$
.

Mesoscopic characteristics of the field are defined as moments of the microscopic density in phase space [11]. Averaging the obtainable stochastic field equations over the entire ensemble of particles yields the obtained equations of the hydrodynamic field (1)-(3), which are hence applicable for describing thermodynamic instabilities in machining processes on both the macroscopic and microscopic structural level.

General Solution. The solution to Eqs. (1)-(3) or constant characteristics of the machined materials, absence of additional energy sources, and constant nonzero speeds and other components of the machining regime has the form

$$\mathbf{v} = \nabla \times \mathbf{v}^* \,. \tag{5}$$

We consider the physical meaning of the solution (5) to Eqs. (1)-(3), which describe the machining process with the velocity fields v of (5). According to the laws of energy and momentum conservation, A = LP, $P = (\rho/\tau)v$, and consequently, with a constant shaping path and constant machining performance of the velocity fields, the momentum and internal energy densities can be described by a dynamic characteristic like the shaping force in machining P.

The divergence of the vector field flow for the force P is described by the expression

$$\nabla \cdot \mathbf{P} = \mathbf{i} \partial P_x / \partial x + \mathbf{j} \partial P_y / \partial y + \mathbf{k} \partial P_z / \partial z , \qquad (6)$$

and the rotation and vortices of the vector field flow are described by the expression

$$\nabla \times \mathbf{P} = \mathbf{i} \left(\frac{\partial P_z}{\partial y} - \frac{\partial P_v}{\partial z} \right) + \mathbf{j} \left(\frac{\partial P_x}{\partial z} - \frac{\partial P_z}{\partial x} \right) + \mathbf{k} \left(\frac{\partial P_v}{\partial x} - \frac{\partial P_x}{\partial y} \right).$$
(7)

The divergence of the flow is observed in the shaping zone in cutting as a result of conversion of the machined material into a chip (Fig. 2). As the ratio of the components of the cutting force P_z/P_y changes (Fig. 2, I) there is a turn of the conventional plane of chip formation to different sides for brittle, low-plasticity materials (Fig. 2a) and ductile, high-plasticity ones (Fig. 2b). This leads to deflections of the chip formation plane as a result of changes in the cross sections of the flows in question (Fig. 2, II), similarly to processes that occur in the motion of a liquid in elbows of tubes of different cross sections. The deflections of the plane of chip formation lead to formation of folds on the free metal surface, whose motions are of a wave character [12]. Motions of the folds owing to a change in the flow velocities at the surface involve formation of vortices near the cutting edge (Fig. 2, III). Similar processes can be observed in the laminar-to-turbulent transition of a liquid flow [13]. The vortices facilitate the motion of the folds and turn the chip formation plane to the previous position. Constantly forming vortices lead to the formation of stagnant dissipative structures in the form of outgrowths on the cutting edge or portions of a step chip that are separated from one another by a localized zone of adiabatic shift (Fig. 2, IV). Processes similar to outgrowth formation occur when a deposit forms in tube elbows as we pass to a larger cross section as a result of a decreased velocity of the liquid flow, while cyclic processes take place as we pass to a smaller cross section owing to an increased flow velocity. The stagnant structures alter the flow cross sections and hence turn the chip formation plane to the previous angle of shift, shifting it periodically by a value equal to the dimensions of the vortex structures.

The above processes in regular orthogonal cutting when x = const and $\partial P_x = 0$ are described by the terms $j\partial P_y/\partial y + k\partial P_z/\partial z$ of expression (6) and represent divergence of the flow in the form of a fan of chip formation surfaces in the YOZ plane. The term $i(\partial P_z/\partial r - \partial P_y/\partial z)$ of expression (7) describes vortices in the formation of



Fig. 2. Schemes of formation of vortex dissipative structures with thermodynamic instabilities in the form of an outgrowth (a) and a step chip (b). Analogies in the motion of liquid flow are shown on the left.

outgrowths when $\partial y \ll \partial z$ and $\partial P_y \ll \partial P_z$ and adiabatic shifts in step chip formation when $\partial y \gg \partial z$ and $\partial P_y \gg \partial P_z$.

We investigate twisting of the chip in forced oblique-angled cutting [14]. By excluding from consideration the YOZ plane and taking z = const and $\partial P_z = 0$ we obtain that the terms $i\partial P_x/\partial x + j\partial P_y/\partial y$ of (6) describe the divergence of the flow as a result of the turn of the chip formation surface in the XOY plane. The term $k(\partial P_y/\partial x - \partial P_x/\partial y)$ of (7) describes the twisting of the chip at high rates of the tool feed and a large angle of slope of the cutting edge when $\partial x >> \partial y$ and $\partial P_x >> \partial P_y$. At small feeds, at a negative value of the angle of slope of the edge, or with the tool's rotation in the direction opposite to the feed when $\partial x << \partial y$ and $\partial P_x << \partial P_y$ the chip twists in the opposite direction.

In cutting with a tool with an additional degree of freedom of the cutting edge in the XOZ plane [6] when y = const and $\partial P_y = 0$, according to the terms $i\partial P_x/\partial x + k\partial P_z/\partial z$ of (6) that describe the divergence of the flow there is a turn of the chip formation surface in the XOZ plane. The term $j(\partial P_x/\partial z - \partial P_z/\partial x)$ of (7) describes free rotation of the cutting edge of the rotational tool under the action of moments of cutting and friction when $\partial x << \partial z$ and $\partial P_x << \partial P_z$ in the direction of rotation of the blank and when $\partial x >> \partial z$ and $\partial P_x >> \partial P_z$ in the direction of chip descent.

NOTATION

Z and z, extensive function of the state of a system and its density; V, volume; r and τ , current coordinates and time; \mathbf{F}_z , local flux density of the quantity Z; \mathbf{q}_z , local strength of the source; ρ , material density; v, flow velocity; e, specific energy; \mathbf{P}^* , tensor of pressure; \mathbf{F}^* , distributed gravity; \mathbf{F}_q and \mathbf{F}_d , heat and diffusion flux densities; T, absolute temperature; s, entropy; F_s , entropy flux density; σ^* , entropy production; N, microscopic density; p and m, momentum and mass of a particle; f, microscopic force determined by all the particles and the external action; v and v_x , v_y , v_z , machining speed and its components; \mathbf{v}^* , component of rotation in a flow moving with the velocity v; A and L, shaping work and path; P and P_x , P_y , P_z , shaping force and its components.

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